LA-5380-MS

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Relativistic Electron Pinch Thermonuclear Device

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UNITED STATES
ATOMIC ENERGY COMMISSION
CONTRACT W-7405-ENG. 36

Printed in the United States of America. Available from
National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151
Price: Printed Copy $4.00; Microfiche $1.45

LA-5380-MS
UC-20
ISSUED: November 1973

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RELATIVISTIC ELECTRON PINCH THERMONUCLEAR DEVICE

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ABSTRACT

A novel plasma pinch device that produces a relatively large nanosecond burst of neutrons and fully stripped ions is being developed at the University of California (Livermore and Berkeley). The electrical efficiency for the burst is in the percent range, and there is evidence that nuclear burn is occurring in the pinch. The device appears to operate like an imploding ion beam, with million electron volt ion temperatures, using the Bennett pinch as a virtual electrode. Reactions observed in a C^{6+},D^{+} plasma produce 10^{11} (D,D) neutrons and a jet of 10^{14} almost monochromatic ions, many of which are C^{6+}. H^{+} and D^{+} at 30 MeV from a 2- to 5-MeV electrode potential show that a collective acceleration process must be invoked.

I. INTRODUCTION

The physical reality of ionic plasma conditions necessary for efficient thermonuclear burn of heavy hydrogen has been approached by Luce and Sahlin[1] of the University of California at Berkeley and Livermore. Inertial plus magnetic confinement of a CD_2 plasma with a 10^{22} cm^{-3} density magnitude, and with a temperature greater than 10^4 eV for about 10^{-9} sec, has produced 10^{11} neutrons per pulse for a 10-kJ electron pulse energy. These data extrapolate to efficiencies for (D,T) in the percent range, suggesting that an increase in particle density of only one or two orders of magnitude is necessary for the success of this approach to nuclear power. The qualitative aspects to this work evolved from a discussion between J. Tuck, Los Alamos Scientific Laboratory (LASL), and J. Luce in 1970, when an alternative was being sought to laser heating of (D,T) pellets for producing thermonuclear energy. Bennett[2] showed that it is possible to use an energetic high-current filament of electrons to heat matter more efficiently than is done with photons. Using this energy in filament-target interactions in a way that is consistent with thermonuclear power production has not been done previously.

II. DESCRIPTION OF THE ELECTRON PINCH

The formation mechanism of an energetic pulse of relativistic electrons with a tens of nanoseconds width and a tens of kilojoules energy is not understood. Bennett found that electron currents much greater than 50 kA could be carried in a plasma sheath on a pencil-sized insulator (with potential loss of only a few hundred kilovolts) and that most of the applied megavolts would be found at the end of the insulator. The high-current filament carrying the energy beyond the end of the rod extends the plasma into a high potential gradient region. Putnam[3] describes the plasma's performance for both the relativistic, high-current filament and its interaction with gases and solids. His general conclusion about the filament was that the ratio of electromagnetic to kinetic energy [shown as ν/γ where $\gamma \equiv \left(1-(v/c)^2\right)^{-\frac{1}{2}}$ and $\nu \equiv Nr_o$ (N ≡ number of electrons/length, $r_o = e^2/m_o c^2$, the classical electron radius)] in the streamer defined its performance. Putnam also concludes that large values of electron pulse energy are possible by using a hollow or partially neutralized charge and a magnetic field to exceed the Lawson[4] current limit (described below). Interaction of this filament with a target (gas or solid) was qualitatively well understood. However, Luce showed that the simple filament-target interaction would produce $\sim 10^7$/pulse neutrons, which is small with respect to magnitudes from his further developments. Another attribute of the relativistic filament resulting in collective acceleration of ions was also discussed by Putnam.[3]

III. EXPERIMENTAL OBSERVATIONS AND PROCESSES

A consistent but qualitative description of the processes in the Sahlin-Luce accelerated mass (Slam) device may be developed from observations made on timing and nucleon yield. Working quickly from electron source to sink is described here.

A 10-ns delay was found between high-voltage onset and electron-current arrival at the ground potential, 5 to 15 cm away. This time is consistent with the formation and progression of a partially neutralized plasma sheath from the cathode along the plasma support rod. Electron current formed by field emission at the rod base is followed by ionization of rod surface material, and radial acceleration of the surface plasma forms a sheath. The electrons move to the anode, normal to the instantaneous equipotentials at the front of the plasma, and the ions move into the space charge limited electron stream. It was shown[5] that a partial neutralization of this plasma suffices to retain the electrons in a strongly electronegative thin sheath. The $n^-/n^+ < \gamma^2$ and the magnetic field, B_θ, around the filament supply the remaining radial restraint on space charge forces on electron trajectory. The ions in this plasma drift[6] rapidly in the $\overline{E}\times\overline{B}$ field toward the cathode: $v_E = c\,\frac{\overline{E}\times\overline{B}}{B^2}$.

At the end of the plasma cathode, the plasma filament is launched into a near-vacuum supported, in part, by ions from the cathode material (CD_2). Further progression of the filament depends upon a supply of ions, since the Lawson limit to total electron current, $I_{max} < 17000\ \gamma v/c$ A, is exceeded by almost an order of magnitude. These ions must come, in small part, from the low-pressure background gas, but largely they come from the anode surface, which has been bombarded for several nanoseconds by relativistic electrons from the filament. According to Putnam, the ionization cross section for D_2 is $\sigma_i \simeq 2 \times 10^{-19}$ cm^2, so by the time the filament arrives, a dense plasma has been formed on the CD_2 anode, and the residual gas in the tank has been ionized. At this time the radius r of the filament can be estimated by equating magnetic field energy and ion kinetic energy, assuming the ions to be accelerated radially across the electric gradient $E = 2Ne/r$ and the magnetic field $B_\theta = 2I/r$. Then $r \approx 0.1$ cm, which is consistent with observation.

The axial progression of the filament from the cathode into the anode slot (diameter $\simeq 1$ cm) carries the cathode potential into a region where the potential gradient is primarily radial in direction. In this transition period, ions are accelerated from the plasma sheath that covers the anode, which is a plate of deuterated polyethylene (CD_2).

The acceleration period of a D^+ ion across a sheath-axial-plasma spacing is $\Delta t \approx 0.3$ ns, which is the period over which the plasma moves into the region of high E_r and low E_z. These ions move from near-ground potential to near-cathode potential at the axis of the filament, and so are very energetic. $\overline{v} \times \overline{H}$ forces also accelerate the ions along the axis away from the cathode. At this time, the circuit has been completed and the electrons in the pinch see the full potential, $\gamma^2 \gg 1$, so $n^-/n^+ \gg 1$, and the energetics of the plasma change from $v/\gamma < 1$ (in the plasma sheath on the rod $v_e \ll c$) to $v/\gamma \ll 1$. The axial filament becomes "rigid" and strongly negative in a manner that depends on time constants, ion supply (to retain $n^-/n^+ < \gamma^2$), and potential gradient distribution. This rigidity is caused by the electron mass increase ($m = m_o\gamma$) and the large increase in electron current due to the increased electric gradient in the filament. As the current increases and the filament radius decreases, $B_\theta\,(I,r)$ greatly increases for small r, and the radial implosion of ions on the anode sheath is radially confined by magnetic and inertial forces.

Until anode sheath implosion, the beam dynamics are consistent with the radial equation of motion reviewed by Putnam.

$$d^2r/dz^2 = -\frac{2}{\beta^2}\,(v/\gamma)\left(f - \frac{1}{\gamma^2}\right) r/a^2 \quad , \qquad (1)$$

where $v = Nr_o = I(A)/17000\beta_z$, N is the number of electrons/length, I = current, $\beta_z c = v_z$, $r_o = e^2/m_oc^2$, a = beam radius, and $f = -\rho^{ion}/\rho^{electron}$, where ρ is the charge density and v_z is the axial velocity of electrons. During the implosion phase, the detailed structure of the pinch becomes complex because of the large effect of collective processes, one of which is the formation of induced axial and radial potential gradients.

The collisionless, charge trajectory determined plasma structure may be visualized as a static "hose" of relativistic (massive) electrons whose diameter is determined by the average curvature radius of the heavy ions accelerated from the anode and from residual gases in the cathode-anode region. This radius of curvature is determined from both the Larmor radius and the deep potential well defined by the magnitude of f in Eq. (1), as modified by $B_\theta(B_\theta^{max} > 10^6$ G). The succeeding series of events, upon arrival at the axial electronic plasma of almost relativistic ions from the anode plasma, is most simply described by the single-particle trajectory theory, using Maxwell's equations. $\nabla X E = -\dot{B}/c$, $\nabla \cdot E = 4\pi\rho$, $\nabla X B = 4\pi j + \dot{E}/c$, and the force equation $F = \rho E + jXB$, with Eq. (1). The arrival of ions at the hose at any point along its boundary changes Eq. (1) to $r^{\sim} \simeq -\frac{2\nu r}{\beta^2\gamma a^2}$, which reduces the hose diameter at a rate determined by ion velocity in a potential gradient $E = 2\rho/r > 10^6$ V/cm where the total charge ρ is a constant. Then, $-\nabla X E = \dot{B}/c = \frac{\partial}{\partial t}\frac{2I}{rc} \simeq -\frac{2I}{r^2c}\dot{r} \to \infty$ as $r \to 0$. This force accelerates ions in +z, electrons in -z directions, and sets up an opposing E(r,z) as $\rho^\pm$ charges separate. Absolute magnitudes are such that the hose is cut, and both ions and electrons are subjected to large oscillating fields at the instantaneous plasma frequency $\omega_p = \sqrt{\frac{4\pi ne^2}{m}}$, where n(t) is determined, in part, by radially oscillating ion density. Collective effects on ρ and $\dot{B}[I(t)]$ are so large that a qualitative picture becomes difficult to see after the first full pinch of the plasma. The relatively inertialess electrons interact by Coulomb collisions and by charge separation forces with the dense ion pinch. The experiment has shown that during and after full implosion, the ions are strongly accelerated by collisional and $\dot{B}$ processes in the z^+ direction, toward a second anode, with an almost pure vector force. This indicates that the magnitude of B_θ is not reduced by reverse currents (due to $\nabla X E = -\dot{B}/c$) until after the major energetics have been established.

The final step in exhausting the electrical energy banks results in the destructive attack of the anode by streamers from the axial plasma. The experimental observation that neutron yield, which results from full plasma implosion and the following radial oscillations of the ions, is inversely proportional to anode damage by these high-current streamers suggests that the instabilities in the axial plasma provide low resistivity current paths by forming filaments containing B (axial in the filament) from B_θ electron acceleration at the surface of the axial (rough) hose plasma. These filaments, which must be formed in pairs (like the two parallel wires of a hairpin), are frequently observed in dense plasma decay, as in the decay of the Marshall plasma gun discharge.

Luce and Sahlin have used only solid targets to obtain an exceedingly dense and high-temperature plasma. Since the initiation of this work in 1970-71, four orders of magnitude in neutron production have been achieved in the state of the art, and one more factor of 10 is expected from electrical network improvement. Theoretical studies to improve the modeling of their work has been useful, but theory cannot at present replace the experimental analyses necessary to include the physics of the Luce-Sahlin device.

IV. OTHER THEORETICAL WORK

Reference 7 reviews the relativistic factors on the equations of physics basic to the electron pulse device, and Refs. 8 and 9 review the Maxwell, force, and energy equations. Babykin's[10] studies of this device are briefly summarized in the Appendix with filament physics studies by Ivanov and Rudakov.[11] Harrison[12] has provided some useful neutron yield estimates, and Erohkin[13] some estimates of potential well magnitudes. Studies by Budker,[5] Rostoker, Bogdankevich, and other Russian scientists are directed toward various aspects of the relativistic beam plasma, particularly with respect to the many instabilities that develop. These instabilities generally dissipate the total energy in too many ways to permit a simple energy transfer to a sufficiently small volume of ions for thermonuclear reaction to occur efficiently. No theory about the collective acceleration of ions in the plasma that result in energy multiplication has been found.

Numerical simulation of the energy distribution, assuming space and time dependence exp $[i(k_y y - \omega t)]$ (Maxwell-Boltzmann distribution of f_o in $f(v) = f_o + f_1(f_o, v, E, B, \omega, T)$ is assumed to make possible a self-consistent solution, does not appear to be an appropriate approach. Also, the fields are neither almost constant, as is usually assumed, nor do they have a simple dependence on one coordinate. Motions are not expected to be bounded, and a relatively small number of plasma oscillations are expected, each one not small with respect to the time-averaged value. None of the numerical methods appear to satisfy the extreme requirements of this plasma analysis. This is now the area of work. These studies do show that the growth of plasma instabilities is sufficiently rapid so that the resulting high fields should play a major part in dissipating the relativistic electron energy. Momentum conservation should transfer a large fraction of electron energy to the ionic plasma, with energy added to the axial ion jet beyond that due to the $v \times H$ and $\dot{B}$ fields.

V. THE EXPERIMENT

Figure 1 shows the Bennett[2] electron source as a simple cylinder tipped with a CD_2 plastic and mounted on a high-voltage electrode.

The electron current is formed by field emission at the base of the CD_2 cathode, and progresses for about 10 ns (10^{-8} sec) into the anode slot where the pinch occurs. Neutrons are formed in a burst with a structure in time suggesting a few damped oscillations. The source of the neutron burst was located by replacing the CD_2 (polyethylene) with CH_2 in three steps. Replacing the cathode or the second anode with CH_2 reduces neutron yield by one-third, replacing the anode reduces the yield by one-twentieth.

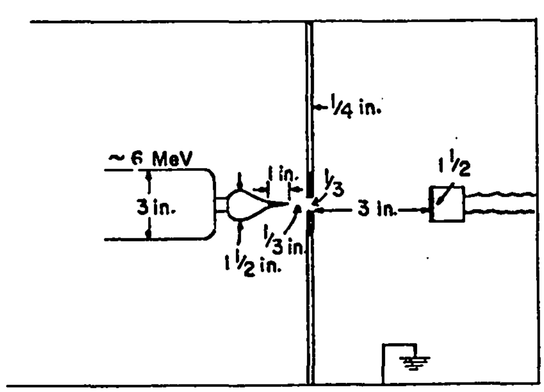

Fig. 1. Geometry of the Luce-Sahlin relativistic pinch device.

The possibility of beam-target interaction as a neutron source was also probed by measuring neutron burst symmetry (it was symmetrical) and by a number of beam-target activation measurements. These measurements showed that C^{6+} was a significant source of target activation. These measurements are not yet available.

The production of (D,T) neutrons from T formed by the D(D,T)n reaction occurred in significant numbers, which further confirms a (D,D) reaction. A significant improvement in neutron yield resulted from texturing the CD_2 plastic surfaces by relatively weak "conditioning" shots. We believe the increase is due to the surface becoming a source of D_2 in the anode slot region. If this is true, and if the mechanism is understood, using a DT microsphere at the point of densest pinch should add a significant factor to the neutron yield. However, the collision period of ions is inversely proportional to Z^4, therefore the thermalizing effect of C^{6+} is large and could be an essential part of the energy-sharing sequence.

Luce noted that as many neutrons were formed by a 10-kJ burst of electrons in his device as were formed by a 500-kJ energy burst in a dense plasma focus device, in which he was previously interested.

The Luce-Sahlin pinch geometry was indicated by the fact that a convex conical second anode was deeply pitted, but with a pit diameter of only $d \approx 0.01$ cm. This dimension is consistent with the qualitative picture drawn in the discussion above.

Recent work has shown what is conceivably a (C,Al) burn, ending in 10^9 atoms of $^{30}Zn^{63}$, in a jet of 10^{13} ions. This Zn isotope could also result from the Zn^{64} (n,2n) reaction, although Zn^{65} would also be expected but was not found.

VI. SINGLE CHARGE DYNAMICS

The initial state of the Luce-Sahlin device is determined by charge trajectories because of the boundary conditions of the pinch. The time constant $t_c \simeq 10^{-10}$ sec, in which time charges are accelerated from the ring surface to the axial streamer, suggests that a qualitative picture of collisionless pinch dynamics can be found by using the particle trajectory code developed by Lewis.[14]

Ion acceleration is started when the space charge field, set up by the axial electron streamer, is relaxed as the electrons collectively move in radially in step with the ion sheath front. In cylindrical coordinates,

$$\nabla\cdot(K\Delta V) \equiv \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(K\rho\frac{\partial V}{\partial\rho}\right) = -4\pi e(n^+ - n^-),$$

and $E = -\partial V/\partial\rho = 2ne/r$ V/cm, where n = charge per unit length at r. If r_o is the stationary charge boundary, $E = 2n_o er/r_o^2$ for n_o charges per unit length. The force due to the magnetic field for $r > r_o$ is $F = Bev/c$, where $B_\theta = 2I/r$. When $r < r_o$, the cancellation of E_r due to the electron space charge at the filament surface reduces the filament radius to that given by charge position r. Then $B_\theta = 2Ir/r_o^2$, where r is the radial position of the ion sheath moving inward, and an axial induction $\nabla X E = \dot{B}/c$ accelerates ions and electrons in the sheath-filament intersection. In cylindrical geometry,

$$\nabla X E = -\partial E_z/\partial r = -\frac{1}{c}\frac{\partial B_\theta}{\partial}\frac{\partial r}{\partial t} = -\frac{v}{c}\frac{\partial B_\theta}{\partial r} = \frac{2I}{r^2}\frac{v}{c} \quad .$$

Then $E_z = 2Iv/c\left(\frac{1}{r} - r/r_o^2\right)$, where the difference is the change in magnetic field before and after charge arrival at the filament radius. For $v_e = c$, the electric force $2ne^2/r$ equals the magnetic force $BeV/c = 2ne^2/r$. Because the electric force is partially neutralized by ions, the lower limit to imploding filament radius is set by collisional and collective forces.

Single-particle trajectories suggest a collapse of the axial pinch to dimensions consistent with magnetic confinement by the azimuthal magnetic field. If we assume charge neutralization ($\rho^- = \rho^+$), then the axial component of the stress tensor for steady state is $\frac{\partial}{\partial z}(-B^2/8\pi + NkT) = 0$. An axial filament of current I and charge number density $N = I/eV$ in volume $V = \pi r^2 l$ for $v = c$ has magnetic pressure $p_m = 4I^2/8\pi r^2$, where $10^9 < p_m < 10^{13}$ dynes/cm^2, consistent with a thermal pressure of the same magnitude only for the static condition. The inertial pressure of the ions arriving behind the shock front adds a component to the stability analysis and to the collective forces. Note that these forces are too small to confine the energetic ions after implosion for a time longer than the time of high-density ion pulse arrival. The relative magnitudes of inertial compression $F = \frac{1}{A}\,d(nmv)/dt \sim NkT$, and the time constant of 10^{-9} sec, are consistent with $NA = 10^{13}$ to 10^{14}.

The ion current to the axis is also space charge limited, with the radial potential in the steady-state condition given by

$$r d^2 V/dr^2 + \partial V/\partial r = 2I\left(\frac{m}{2eV}\right)^{1/2} \quad ,$$

$$\text{so} \quad I = \frac{2}{9}\left(\frac{2e}{m}\right) V^{3/2}/a\beta \text{ amp.},$$

where $\beta(\ell n\ r/a)$.

The total number of ions arriving in 10^{-9} sec is 10^{14}/cm, which is consistent with the anode jet from the fully developed plasma pinch, for $V = 10^7$ V, and the steady-state current $I = 10^4$ A.

The presence of D^+ ions in significant quantity in the volume defined by an anode slot and the filament radius can add a much increased gas target density for the high-energy ions accelerated from the anode surface. The efficiency of energy transformation from electrons to ions and then to thermonuclear burn should be enhanced by sharing the too-high ion energy from the anode with the D_2 gas. The energetics and particle density of the Luce-Sahlin device are consistent with the neutron yield of 10^{11} per burst if the $I = 10^4$ A of multi-MeV ions share energy with the 0.1-torr pressure of D_2 gas. For this condition, the ion-ion collision time $t_c \simeq T^{3/2}/nZ^4 \simeq 10^{-10}$ sec, for $n(D^+) = 5 \times 10^{22}$, is consistent with a high neutron yield Pt_c. The reaction rate $P = n^2\langle\sigma v\rangle V(\text{cm}^3)$ produces 10^{11} neutrons in 10^{-10}sec if $n = 10^{22}$. Luce noted that the efficiency of electrical to thermonuclear yield was proportional to the ratio of field energy to electron kinetic energy. This observation is consistent with the above analysis, if we assume that the energetic massive particles are of two kinds: (1) Those particles accelerated along the z axis by v x B, $\dot{B}$, and momentum transfer, and (2) those particles that share the space charge limited ion pulse energy and participate in a thermonuclear reaction.

VII. SUMMARY

A new device that promises to be either a particle accelerator or a thermonuclear reactor is being developed by Luce and Sahlin. Both methods use collective processes for nanosecond periods and 10^5 A electron current with an electrode potential of 5×10^6 V in a Bennett pinch geometry.

Used as a particle accelerator, the device will produce 30-MeV protons or other ions partially or fully stripped with the same energy-per-unit charge. The number of ions is 10^{14}, and the pulse is almost monochromatic with a small angular divergence.

Used as a thermonuclear reactor, 10^{11} neutrons are produced per pulse, which corresponds to (DT) thermonuclear efficiency in the percent range.

The operating sequence appears to start by forming a plasma filament that accelerates ions radially into a small volume. The filament has a high ratio of electromagnetic to particle kinetic energy, and charge trajectories radially through these fields are consistent with a high positive axial acceleration due to $v_r B$ and $\dot{B}/c$ forces. Charge separation and plasma instability add large but complex forces to the dense ($N = 10^{22}$ ions/cm^3) and energetic ($\bar{V} = 1$ MeV) electronic plasma.

Understanding this device is qualitative because of its essential complexity, small size, and extremely short (10^{-9} sec) period of strongest interaction.

APPENDIX

RELATIVISTIC PINCH STUDIES

I. A STUDY BY BABYKIN[10]

Thermonuclear reaction triggered by an electron beam has been made possible by the development of very energetic electron pulse sources, with a beam in the megampere range. Using collective interactions to transfer this energy to plasma was suggested by Zavoyskiy and Winterberg (quoted in Ref. 10). Harrison showed that the rate of energy absorption needed was of the order of 10^7 to 10^8 J in times less than 10^{-9} sec for an unrestrained plasma. Electron beam pulse formation can be relatively efficient (50%) (Ref. 12) compared to laser energy transfer in this time interval (~0.1%). With the Bennett pinch, the magnetic field associated with this high current beam of electrons provides a natural means of confinement in two dimensions, whereas collective effects offer the possibility of electron energy absorption by an electronic plasma in the third dimension. These electron beams must have small radii if magnetic fields and particle energy density are to be high. The Lawson limit to the electron beam current, $I_{max} = \frac{mc^3}{e} \beta\gamma = 17000\ \beta\gamma$ A, has been exceeded by using a reverse current, or hollow or partially neutralized beams in which $n^-/n^+ > \gamma^2$, where n is charge density, $\beta = v/c$, and $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$. Maximum confining magnetic fields are reduced by plasma current effects. The interaction of the beam with a plasma induces a reverse current when the beam build-up time is less than the skin-depth formation time $\tau = \frac{4\pi\sigma r^2}{c^2}$, where r is the beam radius and σ is the plasma conductivity. The residual magnetic field due to finite conductivity provides a focusing field for the electron stream. The focusing length has been estimated by Ivanov and Rudakov[11] to be

$$\ell = R\left(\frac{\sigma_o p_o}{j'^2_o\, t}\right)^{1/5}\left(\frac{n'_o m' c^2}{p_o}\right)^{1/2}$$

$$\sim I^{1/10}\, t^{-1/5}\, n^{-3/10}\, R^{4/5} \quad ,$$

where $j'_o R$ $\langle\theta\rangle$ is the initial current density, radius, and angle of beam scattering, p_o is the initial plasma pressure, n' and n are the density of the beam and plasma, and t is the injection time. At this length, the beam will be pinched down to a radius $r \simeq \ell\langle\theta\rangle$, where it was assumed that $p' = n'm'c^2\langle\theta\rangle^2$. Anomalously low conductivity may be created at high beam density

$$n'/n > \left(\frac{T_e}{mc^2}\right)^{\frac{1}{2}}$$

when the reverse current may be unstable with respect to the ion oscillation, which will collectively dissipate beam electron energy.

Plasma heating due to the reverse current and the jB_ϕ/c force will scatter the frozen-in magnetic field. The pressure of the plasma in the channel is $nT \simeq n'm'c^2\langle\theta\rangle^2$ where $m' = \gamma m$, therefore simple focusing will occur when the injection time is shorter than the scattering time $t < \frac{r}{c}\left(\frac{mn}{m'n'}\right)^{\frac{1}{2}}$.

As the radial pinch progresses adiabatically, the transverse pressure increases rapidly since $\langle\theta\rangle \sim 1/r$. If the small angle straggling magnitude is also small ($\langle\theta\rangle$ small), then the radius can continue to decrease and electron densities will approach $10^{23}/cm^3$. The energy density would be about 10^{11} J/cm^3 $\left(v \simeq \frac{10^{11} \times 10^7}{1.6 \times 10^{-12}} \frac{1}{10^{23}} \simeq 10^7 \text{ eV}\right)$.

If the electron gas has $T_e \simeq 10^4$ eV and $n_e \simeq 10^{22}/cm^3$, $\nu_{col} \simeq 10^{12}$/sec is less than the frequency $\omega_H = He/mc$. Therefore the gas will not cool off during the characteristic scattering time if

$$\frac{T_o}{\omega_H^2}\frac{\nu_{ei}}{r^2} < \frac{C_s}{r} \quad \text{or} \quad \gamma^2 \langle\theta\rangle^4 > \frac{2Te}{mc^2}\frac{\nu_{ei} r}{C_s}$$

at $T_e \simeq 10^4$ keV, $r = 2 \times 10^{-2}$ cm,

$$C_s \simeq 2 \times 10^7 \text{ cm/sec},$$

$$\text{so } \gamma^2 \langle\theta\rangle^4 > 40 \quad .$$

II. A STUDY BY IVANOV AND RUDAKOV[11]

Intense relativistic electron beams are limited in total energy transport by interaction with the energy's own fields and with the medium in which it is found. For current magnitudes satisfying the Lawson criterion for magnetic forces exceeding electrostatic forces

$$I > I_{crit} = \frac{m_o c^3}{e}\gamma = 17000\beta\gamma$$

where $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$, the beam kinetic energy is less than the magnetic energy, and the electron trajectory has a smaller radius than that of the beam.

The radial contraction of the beam is given by

$$n'\frac{d}{dt} m'v_r = -\frac{\partial p'}{\partial r} - \frac{1}{c} j_z' H_\theta - en'E_r \quad ,$$

and the axial motion is given by

$$n'\frac{d}{dt} m'v_z = -\partial p'/\partial r - \frac{1}{c} j_r H_z + en'E_z \quad ,$$

where n' is the electron number density in the beam, m' is the electron mass, $p' = n'kT'$, $j_z' = n'ew$, $H_\theta \simeq 2I/r$, E_r is determined by the solution of the Poisson equation

$$\frac{\partial E_r}{\partial z} = -\frac{\partial^2 Vr}{\partial_r \partial_z} + \dot{H}_z/c, \quad \text{and}$$

$$\partial E_z/\partial r = -\frac{\partial^2 V}{\partial_r \partial_z} + \dot{H}_\theta/c. \quad \text{Also, } j_z = \sigma(E_z + v_r/c\, H_\theta),$$

and $\partial H_\theta/\partial t = -c\, \partial/\partial_r\, (j_z'/\sigma + v_r/c\, H_\theta)$ where $\sigma \simeq 10^{13}\,[T(eV)]^{3/2}\,sec^{-1}$. Ivanov and Rudakov[11] have shown that relativistic electrons in a plasma break up into thin filaments with average $B_z \simeq 0$.

III. A YIELD STUDY BY HARRISON[12]

Particles accelerated into a volume react with a rate given by $n^2\langle\sigma v\rangle$ and radiate with a rate given by $k n^2 T^{\frac{1}{2}}$ erg/cm³-sec with $k \simeq 5 \times 10^{-25}$ given by Spitzer $\langle\sigma v\rangle_{DD} = 1.4 \times 10^{-17}$, $\langle\sigma v\rangle_{DT} = 3.3 \times 10^{-16}$ for $T_{DD} \approx 4 \times 10^8$ and $D_{DT} \simeq 2 \times 10^{8}$°K. Then radiative energy losses approximately equal reaction gain for $(\rho D)_{DD} \simeq 1$ and $(\rho D)_{DT} \simeq 0.03$ for these two mixtures which are shock heated and inertially confined. Adding the confining forces of the magnetic field for small potential gradients has been shown by Ivanov and Rudakov[11] to add an equilibrium radius, ℓ, equal to the focusing length above. The derivation shows that the reverse current in a plasma tends to dissipate the confining magnetic field, therefore injecting an electron beam into a plasma must be avoided as a technique for plasma ignition. The exception is when a large reverse current along $\vec{F} = \vec{E}\times\vec{B}$ can cause charge separation and a strong oscillation along $\vec{F}$. If B can be excluded from the plasma volume, this may more rapidly transfer electron to ion energy due both to Coulomb collisions and ion acceleration.

IV. A STUDY BY EROHKIN ET AL.[13]

The interaction of particles and waves may result in capture of the beam particles in potential wells of the wave. The characteristic time of the particle oscillation is

$$T_o \simeq 1/k_o\sqrt{e\theta/m} \quad ,$$

where $k_o \equiv c/\lambda$ is the wave number and θ is the potential amplitude in the wave. If beam particle velocities have a widespread $\Delta u/u_o \gg (n_1/n_o)^{1/3}$ where u is velocity and n_1 and n_o are beam and plasma densities, the beam increment is linear wit

$$\gamma_L = (2\pi^2e^2/mk^2)\ \omega_p\ \partial f_o/\partial v \quad .$$

Here, f_o is the equilibrium distribution function and $v_\theta = \omega_p/k$ is the phase velocity of the wave.

The linear approximation for plasma thermal particles is

$$\frac{dv}{dt} = -\frac{e}{m}E(t)\ \sin k_z,\ \frac{dz}{dt} = v$$

$$\frac{1}{4\pi}\ E(t)\ \frac{dE}{dt} = -\ \overline{j^{res}\ E(t,z)}$$

$$= \frac{eE(t)}{\lambda}\int_{-\lambda/2}^{+\lambda/2} dz_o \int_{-v_o^m}^{v_o} dv_o\ f_o(v_o+v_\theta)$$

$$\sin k_z\ (v+v_\theta) \quad .$$

The current of resonant particles is j^{res} and $dzdv = dz_o dv_o$ was used for conservation of phase space volume, as well as the constant distribution function over the trajectory.

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